Abstract

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The p-Laplace evolution equation as p goes to 1: Toward a general convergence result for parabolic minimizers

In this talk, we shall present some recent results on the convergence of minimizers of convex functionals and (parabolic) variational inequalities, occurring e.g. in material science and plasticity, image processing, minimal surfaces, nonlinear spectral theory and harmonic analysis. The main topic of investigation is the so-called *Mosco convergence* of integral-type functionals satisfying certain weak growth and coercivity bounds. By definition, Mosco convergence generates a finer topology than the well-known Γ-*convergence*, and can be verified in various applications to very well suit the needs of convergence problems of parabolic variational minimizers, as there are e.g. sensitivity, continuous dependence on parameters in the related PDEs and periodic homogenization.

Following some ideas from [T., *Preprint* (2011)], we establish the Mosco convergence for linear and p-growth functionals with Dirichlet boundary terms on Lipschitz domains $\Omega \subset \mathbb{R}^n$ in the spaces $L^1(\Omega;\mathbb{R}^N)$ and $L^p(\Omega;\mathbb{R}^N)$ respectively. The key-a priori estimate in the proof follows by an approximation result originating from De Giorgi (1968). A BV-integration by parts formula by Graziani (2009) provides useful to analyze the convergence, and finally, duality results from [Beck and Schmidt, *J. Funct. Anal.* (2015)] help to identify the limit and to find good approximations.

In the end of the talk, we shall sketch some open problems related to parabolic variational inequalities in the spirit of [Bögelein, Duzaar and Marcellini, *J. Differential Equations* (2014)] for which, from a naive viewpoint, Mosco convergence seems to be not quite sufficient for convergence of solutions; rather than that, Mosco convergence plus some additional knowledge about the good approximations seems to be needed. We note that, in order to avoid a similar technical issue, the notion of *random Mosco convergence* was introduced in [Gess and T., *J. Differential Equations* (2016)], where convergence of stochastic evolutions via variational inequalities is studied.