VARIABILITY OF PATHS AND DIFFERENTIAL SYSTEMS WITH BV-COEFFICIENTS

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Abstract. We study existence and regularity of generalized Lebesgue-Stieltjes integrals

$$\int_0^t \varphi(X_s)\, dY_s, \quad t \geq 0,$$

where X is a multidimensional Hölder continuous path, Y is a Hölder continuous driving path and φ is a function of (locally) bounded variation. We shall give a meaningful definition for the compositions $\varphi(X)$ and prove, using certain fine properties of BV-functions, that they are sufficiently regular for the above integral to make sense.

The key idea to manage this is a relative and quantitative condition between the coefficient φ on the one hand and the path X on the other hand. This condition ensures that the path X spends very little time in regions where the coefficient is particularly irregular, and is made precise and discussed systematically in terms of mutual Riesz energy of the occupation measure of the path X and the gradient measure of the coefficient function φ, where we shall provide sufficient conditions and examples in terms of upper regularity estimates for Borel measures. Examples including X being a typical path of a fractional Brownian motion and φ being an indicator function of a set of finite perimeter are discussed.

Furthermore, we shall prove a change of variable formula and, given slightly higher regularity, provide a quantitive approximation scheme by Riemann-Stieltjes sums.

Under further conditions, we also establish existence, regularity and uniqueness results for Hölder continuous solutions to systems of differential equations determined by integrals of the above type.

The talk is based on a joint work with Michael Hinz (Universität Bielefeld) and Lauri Viitasaari (Aalto University), see `https://arxiv.org/abs/2003.11698`.